Sovereign Default Risk and Uncertainty Premia

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Motivation

- Sovereign default risk is key for high and countercyclical interest rates.
- Arellano (2008) was able to qualitatively explain most of these empirical facts.
- Quantitatively, it fell short in its asset-pricing implications, including explaining high bond spreads for historical levels of default frequency:

Argentina 1983:2001

Arellano:	3.58%
Data:	10.25%

- A similar puzzle has been reported in the corporate finance literature (e.g. see Huang and Huang (2003)).

This paper...

- General equilibrium model with uncertainty-averse lenders and endogenous default that:
  - explains dynamics of bond spreads observed for Argentina while keeping default frequency at historical levels
  - replicates standard empirical regularities of emerging economies
- Plausible degrees of risk aversion on lender's side alone with standard expected utility are not sufficient to generate high spreads
- Propose new moment-based uncertainty measure to gauge amount of belief distortion at the lower tail of probability distributions.

Review of Literature

- **Business cycles in emerging economies:** Neumeyer and Perri (2005), Uribe and Yue (2006)
- **Sovereign Default:**
  - Eaton and Gersovitz (1981), Arellano (2008), Aguiar and Gopinath (2006)
  - Lizarazo (2010), Borri and Verdelhan (2009), Costa (2009)
  - Arellano and Ramanarayanan (2010), Hatchondo, Martinez and Sosa-Padilla (2010)
  - Chatterjee and Eyigungor (2009), Hatchondo, Martinez and Sapriza (2010)
- **Robust Control:** Hansen and Sargent (2007a, 2007b)
- **Peso problem and rare events:** Barro (2006), Nakamura, Steinsson, Barro and Ursua (2011),

Benchmark Model with One-Period Debt

- Risk-averse borrower interacts with continuum of identical foreign lenders.
- Financial markets are incomplete: only a one-period discount bond can be traded; it can be defaulted upon.
  $\longrightarrow$ *Debt contract:* (q, B')
- Borrower's endowment y follows a Markov process.

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Arellano (2008)

- Risk-neutral lenders with rational expectations
- Equilibrium bond prices?

$$q(y, B') = \frac{1 - \Pr(\text{Default}(y', B'))}{1 + r^f}$$

Strong link between prices and actual default probability

Our Benchmark Model

- Uncertainty-averse lenders
- Equilibrium bond prices?

$$q(y, B') = \frac{1 - \widetilde{\Pr}(\text{Default}(y', B'))}{1 + r^f}$$

Strong link between prices and distorted default probability

Key Insight with One-Period Debt

Payoff

$$\delta_{t+1} = \begin{cases} 1 & \text{if borrower repays} \\ 0 & \text{if borrower defaults} \end{cases}$$

$$\begin{aligned} q_t &= E_t(SDF_{t+1}\delta_{t+1}) \\ &= \underbrace{E_t(SDF_{t+1})}_{\frac{1}{1+r^f}} \underbrace{E_t(\delta_{t+1})}_{(1-\text{Pr}_t(\text{Default}))} + \underbrace{Cov_t(SDF_{t+1}, \delta_{t+1})}_{<0} \end{aligned}$$

Outline of the Talk

- Model with Long-Term Debt
- Recursive Equilibrium
- Numerical Results
- Uncertainty Measures
- Conclusion

Borrower's Problem

Total output is given by $y + x$, where

- y follows Markov process with cond. density $f(y'|y)$
- x is i.i.d.

  $\rightarrow$ x is introduced to guarantee convergence with long-term debt.

Let $w = (x, y)$.

Long-term debt is modeled as in Chatterjee and Eyigungor (2012):

- A bond matures next period with probability λ,
- If it does not mature, it pays a coupon ψ.

Borrower's Problem

Two stages the borrower can be in:

- *financial autarky*
- access to international financial markets.

In case of default, the borrower incurs two types of costs:

- (temporary) exclusion from financial markets
- direct output cost

Zero recovery rate of defaulted debt.

Borrower's Problem: Access to Financial Markets

$$V(w,B) = \max\left\{\underbrace{V_A(\underline{x},y)}_{\text{value of autarky}} \quad , \quad \underbrace{V_R(w,B)}_{\text{value of repayment}}\right\}$$

Default decision $\qquad \delta(w,B) \;=\; \mathcal{I}\left\{V_A(\underline{x},y) \leq V_R(w,B)\right\}$

Borrower's Problem: Repaying Debt and Financial Autarky

If borrower decides to repay

$$
\begin{aligned}
V_R(w, B) &= \max_{c, B'} \left\{ U(c) + \beta E\left[V(w', B') \right] \right\} \\
s.t.\ c &= y + x - q(y, B')(B' - (1-\lambda)B) + (\lambda + (1-\lambda)\psi)B
\end{aligned}
$$

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In financial autarky

$$V_A(w) = U(y + x - \phi(y)) + \beta E\left[(1-\pi) V_A(w') + \pi V(w',0) \right]$$

Lender's Problem: Preferences

- Lenders acknowledge that *approximating model* f could be potentially misspecified.
- They consider a set of alternative conditional densities $\widetilde{f}$ and seek decision rules that are robust to it.
- Why?
  - limited availability of official reliable data
    (e.g. see Chuhan (1992), Boz, Daude and Durdu (2011))
  - measurement errors
  - lags in release of official statistics with subsequent revisions

Lender's Problem: Preferences

Endow lenders with **multiplier preferences**.

Let $m_{t+1}(y_{t+1}|y^t) \equiv \begin{cases} \frac{\widetilde{f}(y_{t+1}|y_t)}{f(y_{t+1}|y^t)} & \text{if } f(y_{t+1}|y_t) > 0 \\ 1 & \text{if } f(y_{t+1}|y_t) = 0 \end{cases}$

$$W(w, B, b) = \max_{c^L, b'} \min_{m' \geq 0} \left\{ c^L + \gamma E_Y \left[m' E_X W(y', B', b') + \theta m' \log m' \right] \right\}$$

$$s.t. \quad E_Y m' = 1$$

where θ is a penalty parameter that measures the degree of concern about model misspecification.

Lender's Problem: Risk-Sensitive Operator

Solving for m' leads to

$$W(w, B, b) = \max_{c^L, b'} c^L + \gamma T^\theta \left[E_X W(w', B', b') \right]$$

where $T^\theta(\cdot)$ is the risk-sensitive operator given by

$$T^\theta \left[E_X W(w', B', b') \right] \equiv -\theta \log \left(\int \exp \left\{ -\frac{E_X W(x', y', B', b')}{\theta} \right\} f(y'|y) dy' \right)$$

Lender's Problem

If the borrower decides to repay

$$\begin{aligned} W_R(w,B,b) &= \max_{c^L,b'} \left\{ c^L + \gamma T^\theta \left[E_X W(w',B',b') \right] \right\} \\ s.t.\ c^L &= \overline{y}^L + q(y,B')(b' - (1-\lambda)b) - (\lambda + (1-\lambda)\psi)b \\ B' &= \Gamma^L(w,B) \end{aligned}$$

with $W(w',B',b') \equiv \delta(w',B')W_R(w',B',b') + (1-\delta(w',B'))W_A(y')$

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In financial autarky

$$W_A(y) = \bar{y}^L + \gamma T^\theta[(1-\pi)W_A(y') + \pi E_X W_R(w',0,0)]$$

Recursive Equilibrium

A recursive equilibrium is given by policy functions for consumption, bond holdings, default and probability distortions, bond prices, perceived laws of motion for bond holdings and value functions, for borrower and individual lender, such that:

1. Given prices and perceived laws of motion for debt holdings, policy functions and value functions solve the borrower and individual lender's problems.

2. Given the lender's policy functions and value functions, the probability distortions solve the minimization problem.

3. Bond prices clear the financial markets

$$B'(w,B) + b'(w,B) = 0$$

4. The actual and perceived law of motions for debt holdings coincide:

$$B'(w,B) \quad = \quad \Gamma^L(w,B)$$

Equilibrium Bond Prices

$$q(y, B') = \gamma \int_{\mathbb{Y}} \chi(y', B') \underbrace{m_R^*(y'|y, B') f(y'|y)}_{\tilde{f}(y'|y, B')} dy'.$$

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where

$$\chi(y', B') = E_X \left[(\lambda + (1 - \lambda)(\psi + q(y', B'(w', B')))\delta^\star(w', B') \right]$$

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$\theta = +\infty \qquad SDF = \gamma$

$\theta < +\infty \qquad SDF = \gamma m_R^*(y'|y, B')$

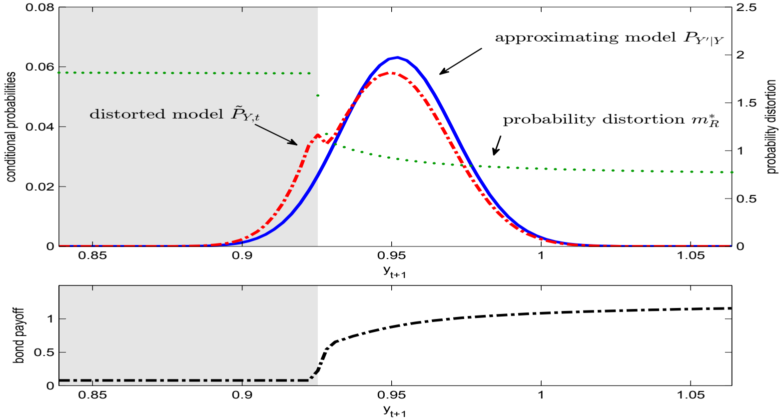

Figure: Approximating density, distorted density, and probability distortion.

Note: $y_t = e^{-0.5\sigma_\varepsilon}$, $x_t = 0$ and B_t is set to the median of its unconditional distribution.

Approximating Model and Distorted Model

Moment	Approximating Model	Distorted Model
Mean(y_{t+1})	0.9518	0.9481
Std.dev.(y_{t+1})	0.0191	0.0202
Skewness(y_{t+1})	0.0601	0.0811
Kurtosis(y_{t+1})	3.0064	2.7910

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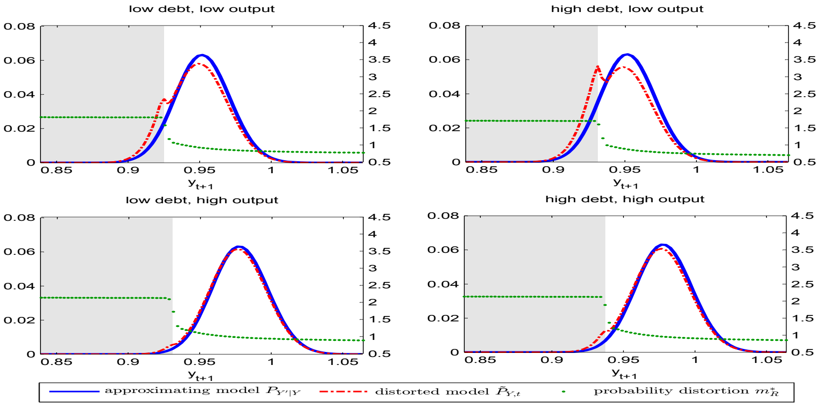
Conditional probability of default:

- under $\tilde{f}$: 16.2% (quarterly).
- under f: 9.3% (quarterly).

The solution to the robust control problem entails putting **more probability mass** to the default event.

Probability distortions are state specific...



Note: low B_t is set to 50th percentile of its unconditional distribution and high B_t to 60th percentile. Low $y_t = e^{-0.5\sigma_\varepsilon}$, and high $y_t = e^{-0.25\sigma_\varepsilon}$. $x_t = 0$.

Other Useful Results

Lemma 1: Consider a RCE for given $\hat{y}^L$. For any other $\tilde{y}^L$ sufficiently large, there exists a RCE with same bond prices and borrower's allocations.

Let $(1 + r^f)\gamma = 1$.

Lemma 2: Consider an economy where lenders can borrow or save at gross risk-free rate $1 + r^f$ from international capital markets. For any RCE in this new economy, there exists an RCE with zero risk-free asset holdings and identical equilibrium prices and borrower's allocations.

Penalty Parameter θ

Assume that the lender is concerned about models that are difficult to distinguish from each other *given available data.*

The penalty parameter θ is context specific.

To measure the amount of probability distortion, we use

i. **Detection error probabilities (DEP)**

   $\rightarrow$ consider likelihood ratio tests for distinguishing the approximating model (A) from the worst-case model (W) associated to some θ.

ii. **Moment-based uncertainty measure**

   $\rightarrow$ allows us to focus on lower tail of distributions, where the discrepancy between models is larger.

Detection Error Probabilities

$DEP_T \in [0, 0.5]$.

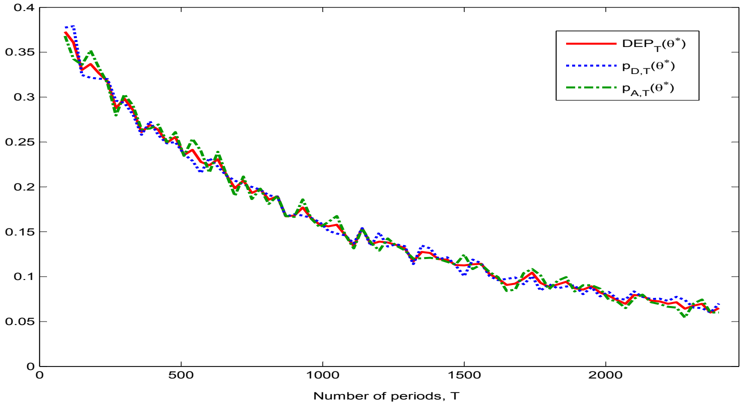
- $DEP_T = 0$: A and W are very different, i.e., no mistakes are made.
- $DEP_T = 0.5$: A and W are the same.

What is an "acceptable value"?

- ≥ 0.2 for Barillas, Hansen and Sargent (2009).
- ≥ 0.1 for Maenhout (2005), Anderson, Hansen and Sargent (2008), Dreschler (2011).

In our case, for $\theta = 0.619$, $DEP_{240} = 0.313$.

Detection Error Probabilities



Numerical Simulations: Functional Forms

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

$$\phi(y) = max(0, \kappa_1 y + \kappa_2 y^2) \qquad \text{with } \kappa_2 > 0$$

$$\ln y_{t+1} = \rho \ln y_t + \sigma_\varepsilon \varepsilon_{t+1}$$

$$\varepsilon_{t+1} \sim i.i.d.\mathcal{N}(0,1)$$

Parameter Values

		Parameter	Value
Borrower	Risk aversion	σ	2
	Time discount factor	β	0.9627
	Probability of reentry	π	0.0385
	Output cost parameter	κ_1	-0.255
	Output cost parameter	κ_2	0.296
	AR(1) coefficient for y_t	ρ	0.9484
	Std. deviation of ε_t	σ_ε	0.02
	Std. deviation of x_t	σ_x	0.03
Lender	Robustness parameter	θ	0.619
	Constant for z	$\log(\overline{z})$	1.00
Bond	Risk-free rate	r^f	0.01
	Decay rate	λ	0.05
	Coupon	ψ	0.03

Calibration

Statistic	Data	CE Model	Baseline Model	Our Model
Mean$(r - r^f)$	8.15	8.15	5.01	8.15
Std.dev.$(r - r^f)$	4.58	4.43	4.27	4.62
mean$(-b/y)$	46	70	42	44
Std.dev.(c)/std.dev. (y)	0.87	1.11	1.16	1.23
Std.dev.(tb/y)	1.21	1.46	0.89	1.23
Corr(y, c)	0.97	0.99	0.99	0.98
Corr$(y, r - r^f)$	-0.72	-0.65	-0.78	-0.75
Corr$(y, tb/y)$	-0.77	-0.44	-0.80	-0.68
Drop in y (around default)	-6.4	-4.5	-3.9	-5.6
DEP	*NA*	*NA*	50.0	31.3
Default freq. (annual)	3.00	6.60	3.00	3.00

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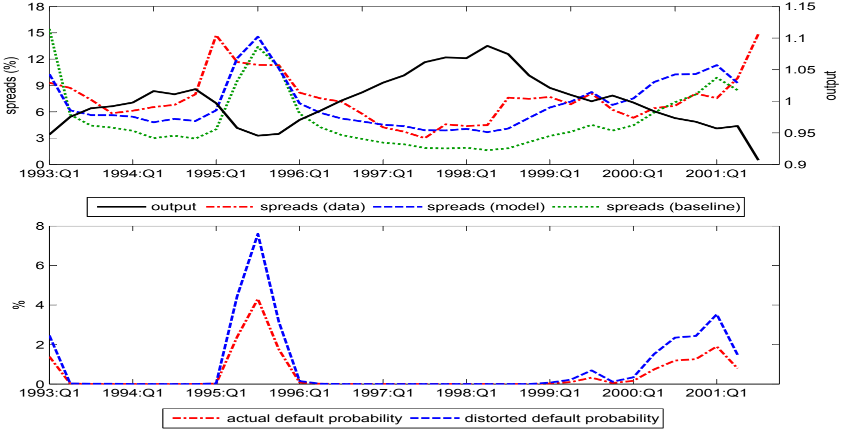
BC for Argentina: Our Model, Data and Baseline Model



MC Results: Different Degrees of Robustness

Statistic	$\theta = +\infty$	$\theta = 5$	$\theta = 1$	$\theta = 0.75$	$\theta = 0.5$	$\theta = 0.25$
Mean$(r - r^f)$	4.54	4.83	6.43	7.23	9.30	18.74
Std.dev.$(r - r^f)$	3.32	3.41	3.96	4.28	5.10	9.36
Mean$(-b/y)$	43.32	43.40	43.93	43.96	43.78	42.22
Std.dev.(c)/std.dev.(y)	1.17	1.18	1.21	1.22	1.23	1.22
Std.dev.(tb/y)	0.86	0.92	1.11	1.17	1.28	1.40
Corr(y, c)	0.99	0.99	0.98	0.98	0.98	0.97
Corr$(y, r - r^f)$	−0.79	−0.78	−0.77	−0.76	−0.74	−0.67
Corr$(y, tb/y)$	−0.77	−0.76	−0.72	−0.70	−0.66	−0.55
DEP	0.50	0.469	0.377	0.344	0.267	0.095
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Corr(y, c)	0.99	0.99	0.98	0.98	0.98	0.97
Corr$(y, r - r^f)$	−0.79	−0.78	−0.77	−0.76	−0.74	−0.67
Corr$(y, tb/y)$	−0.77	−0.76	−0.72	−0.70	−0.66	−0.55
DEP	0.50	0.469	0.377	0.344	0.267	0.095
Default freq. (annual)	3.00	3.00	3.00	3.00	3.00	3.00

MC Results: Different Degrees of Robustness

Statistic	$\theta = +\infty$	$\theta = 5$	$\theta = 1$	$\theta = 0.75$	$\theta = 0.5$	$\theta = 0.25$
Mean$(r - r^f)$	4.54	4.83	6.43	7.23	9.30	18.74
Std.dev.$(r - r^f)$	3.32	3.41	3.96	4.28	5.10	9.36
Mean$(-b/y)$	43.32	43.40	43.93	43.96	43.78	42.22
Std.dev.(c)/std.dev.(y)	1.17	1.18	1.21	1.22	1.23	1.22
Std.dev.(tb/y)	0.86	0.92	1.11	1.17	1.28	1.40
Corr(y, c)	0.99	0.99	0.98	0.98	0.98	0.97
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Corr$(y, tb/y)$	−0.77	−0.76	−0.72	−0.70	−0.66	−0.55
DEP	0.50	0.469	0.377	0.344	0.267	0.095
Default freq. (annual)	3.00	3.00	3.00	3.00	3.00	3.00

MC Results: Risk Aversion and $\theta = +\infty$

Statistic	$\sigma^L = 1$	$\sigma^L = 2$	$\sigma^L = 5$	$\sigma^L = 10$	$\sigma^L = 20$	$\sigma^L = 50$
mean$(r - r^f)$	4.18	4.20	4.20	4.25	4.37	5.36
std.dev.$(r - r^f)$	3.66	3.72	3.63	3.69	3.90	11.72
mean(r^f)	6.93	6.85	6.85	4.86	-0.52	-30.78
std.dev.(r^f)	0.34	0.68	1.69	3.34	6.41	11.72
mean$(-b/y)$	79.33	79.04	78.08	79.20	86.31	118.88
std.dev.(c)/std.dev.(y)	1.84	1.83	1.84	1.87	2.05	3.78
std.dev.(tb/y)	7.38	7.32	7.29	7.52	8.56	18.38
corr(y, c)	0.74	0.73	0.73	0.73	0.71	0.57
corr$(y, r - r^f)$	-0.55	-0.55	-0.56	-0.55	-0.54	-0.46
corr$(y, tb/y)$	-0.28	-0.28	-0.28	-0.28	-0.29	-0.30
Default frequency	3.00	3.00	3.00	3.00	3.00	3.00

$$\ln c_{t+1}^L = \rho^L \ln c_t^L + \sigma_\varepsilon^L \varepsilon_{t+1}^L$$

$$\varepsilon_{t+1}^L \sim i.i.d.\mathcal{N}(0,1)$$

Concluding Remarks

- Model uncertainty is key to explain the dynamics of sovereign bond spreads in emerging economies while preserving a low default frequency.
- It provides micro-foundations for *ad hoc* pricing kernels used in the sovereign default literature.
- Significantly lower amount of model uncertainty is necessary relative to equity premium models.
- Future research:
  - introduce secondary markets for defaulted bonds
  - analyze "contagion" effects.